On the anisotropic elastic properties of woods

J. Lawrence Katz · Paulette Spencer · Yong Wang ·
Anil Misra · Orestes Marangos · Lisa Friis

Abstract In 2003 Nature Materials article, Keckes et al. presented deformation properties of a variety of woods in relation to deformation of their individual wood cells. Their point is "The remarkable mechanical properties of biological materials reside in their complex hierarchical structure…". This holds for mineral-based biological materials such as bone as well as for wood. Indeed, one of us (J.L.K.) introduced the concept that to explain the material properties of cortical bone, it was necessary to treat it as a complex material/structural hierarchical composite. Calculations to determine anisotropic properties of bone measured using ultrasonic wave propagation techniques, were extended to similar measurements on both soft and hard woods. These anisotropic properties calculations have been extended to include data based on mechanical measurements of orthotropic elastic constants of both soft and hard woods for comparison with both earlier ultrasonic measurements and mechanical testing on other woods. This work illustrates the fact that understanding and modeling the properties of wood is a complex task as the symmetry changes with scale. For example, lignin is isotropic, hemicellulose and cellulose are transversely isotropic, while the cells and microstructure have orthotropic symmetry.

J. L. Katz (✉) · P. Spencer · Y. Wang
Department of Oral Biology, School of Dentistry, University of Missouri-Kansas City-Center for Research on Interfacial Structure and Properties (UMKC-CRISP), University of Missouri-Kansas City, 650 East 25th Street, Kansas City, MO 64108-2784, USA
e-mail: katzjl@umkc.edu

J. L. Katz · A. Misra · O. Marangos
Division of Civil & Mechanical Engineering, School of Computing & Engineering, University of Missouri-Kansas City, Kansas City, MO, USA

L. Friis
Department of Mechanical Engineering, University of Kansas, Lawrence, KS, USA

Introduction

Wood has been an important structural material for millennia. Its properties have intrigued engineers and forestry scientists to the point that a considerable literature on structure–property relationships developed post-World War 2. This includes, among others: The Elasticity of Wood and Plywood [1]; chapter 20 in The Testing of Engineering Materials (4th edition) [2]; the 4 volume EMMSE/MEC set: vol. I, Wood: Its Structure and Properties [3]; vol. II, Wood as a Structural Material [4]; vol. III, Adhesive Bonding of Wood and Other Structural Materials [5]; and vol. IV, Wood: Engineering Design Concepts [6]. Also in this category are: chapter 4 in Wood Handbook: Wood as an Engineering Material [7] and Acoustics of Wood [8].

Keckes et al. in 2003 study presented the deformation properties of a variety of woods in relation to the deformation of their individual wood cells [9]. They make the point that "The remarkable mechanical properties of biological materials reside in their complex hierarchical structure…". This holds for mineral-based biological materials such as bone as well as for wood. Indeed, one of us (J.L.K.) introduced the concept that in order to explain the material properties of compact haversian cortical bone, it was necessary to treat it as a complex material/structural hierarchical composite [10, 11]. The insight that led Katz to put forth this concept for modeling bone's material
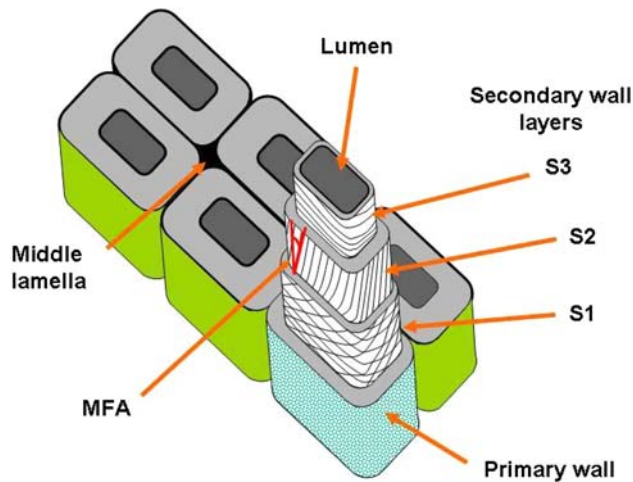
Fig. 1 Kretschmann: Wood Cells Hierarchical Anatomy ([18] with permission of Nature Materials)

properties evolved from a series of ultrasonic wave propagation (UWP) studies of the anisotropic elastic properties of cortical bone [12–15] following a similar ultrasonic study of the anisotropic properties of fluoroapatite [16] and the modeling of the anisotropic elastic constants of hydroxyapatite [17], the inorganic ("mineral") component of bone.

Kretschmann [18] following on the Keckes et al. work [9], describes the "complicated hierarchical structure" of wood including an illustration of wood's anatomy (Fig. 1) [18]. A comparison of this illustration with one of several often used as descriptive of the hierarchical structure of bone (Fig. 2) [19] shows the remarkable similarity of the micro- and ultra-structural aspects between both, except that on the microstructural level, haversian bone is based on circular cross sections (transverse isotropy) while on the cellular level that for wood is based on rectangular cross sections (orthotropy).

The Keckes et al. [9] and Kretschmann [18] articles symbolize what appears to be a most recent resurgence of interest in the properties of woods, see for example, in 2003, Fracture and Fatigue in Wood [20]; additionally in

2004: chapters 1, 2 (in part), 3, 4, and 8 (in part) in Optimization Mechanics in Nature [21]; a paper in May 2004 MRS Bulletin [22]; an article in July 2004 Mechanical Engineering [23]; a paper in the 2004 European Journal of Physics [24]; and continuing into 2005 with a Paper just accepted by the European Journal of Mechanics A: Solids [25].

Elastic anisotropy

Both the Keckes et al. [9] and Kretschmann [18] discourses are based on quasi-static mechanical testing of the uniaxial tensile stress–strain behavior. Enhancement of the understanding of the deformation properties of wood necessitates a description of its anisotropic properties as well. Clearly, both wood and bone must have considerable transverse elastic properties as well as those along the longitudinal direction, i.e., along the species' principal axis. Obtaining the full set of orthotropic elastic stiffness coefficients for both bovine plexiform cortical bone [26] and soft and hard woods [27, 28] also allows calculation of their directional traditional elastic properties, Young's, bulk and shear moduli and Poisson's ratio, as well as their Voigt and Reuss moduli and the shear and compressive scalar anisotropy factors [29]. The Voigt modulus is obtained by averaging the stiffness coefficients over all possible crystal (or equivalent texture) orientations in the material; the Reuss modulus is obtained by averaging the compliance coefficients over all possible crystal (or equivalent texture) orientations in the material. Therefore, the Voigt modulus represents the upper bound on the elastic properties, where a uniform strain distribution is assumed, and the Reuss modulus represents the lower bound on the elastic properties, where a uniform stress distribution is assumed. Such sets of orthotropic elastic constants for bovine plexiform cortical bone and soft and hard woods are given in Tables 1 and 4, respectively, in Katz and Meunier [30]. The formulae to calculate the Voight and Reuss values are provided in the appendix of that paper [30].

Fig. 2 Lakes: Bone Hierarchical Anatomy ([19] with permission of Nature)
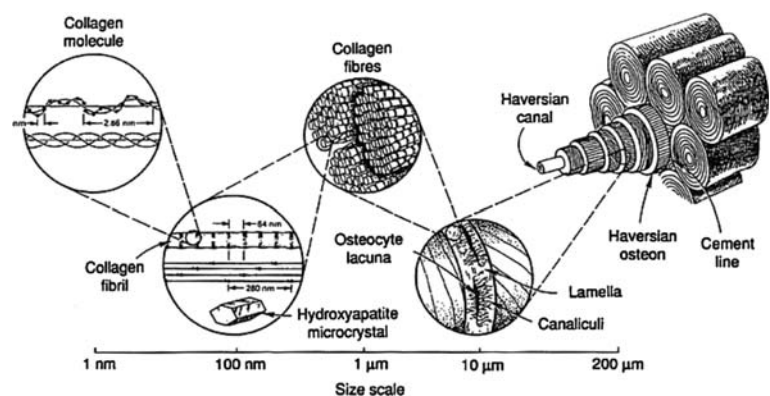
The Voigt and Reuss moduli are important in this case because they are the bases for the Chung and Buessem calculation [30] adapted for the present analyses. As Chung and Buessem point out "…for an aggregate containing anisotropic crystals, a difference between the limiting values is expected from the Voigt and Reuss assumptions. The concept underlying the present theory of the Percent Elastic Anisotropy is that the magnitude of this difference is a function only of the degree of elastic anisotropy possessed by the crystal under consideration."

In order to compare these disparate ultrasonic wave propagation (UWP) measurements of anisotropic elastic constants as well as those done by mechanical testing, Katz and Meunier [29, 30] adopted the calculation by Chung and Buessem [31] for single crystals of various inorganic materials with either cubic or hexagonal (transverse isotropic) symmetry. Chung and Buessem provided the rationale for the Percent Elastic Anisotropy calculation in the opening two sentences of their Abstract, "The elastic anisotropy of a crystal is the orientation-dependence of the elastic moduli or sound velocities. Essentially, all the known crystals are elastically anisotropic." They go on to say that their "…paper presents a convenient method of describing the degree of elastic anisotropy in a given cubic, hexagonal, trigonal, and tetragonal crystal…". The calculation yields two scalar anisotropy factors, one for shear anisotropy, As* (%), and one for compressive anisotropy Ac* (%), based on algebraic relationships between the full set of elastic constants, five C_{ij} for transverse isotropy (hexagonal symmetry in crystal physics):

$$\text{As}^*\,(\%) = \frac{G^{\text{V}} - G_{\text{R}}}{G^{\text{V}} + G_{\text{R}}} \quad \text{and} \quad \text{Ac}^*\,(\%) = \frac{K^{\text{V}} - K_{\text{R}}}{K^{\text{V}} + K_{\text{R}}} \quad (1)$$

where G is the shear modulus and K is the bulk modulus.

This calculation was adapted by Katz and Meunier for compact haversian cortical bone exhibiting transverse isotropic symmetry [30] (hexagonal symmetry in crystal

physics) based on earlier modeling [11] and ultrasonic measurements [13, 14] that showed that selected volumes of compact bone could be treated as pseudo-crystals on the microstructural level. They then extended the calculation to cover the full set of nine C_{ij} for orthotropic symmetry [29] (orthobombic symmetry in crystal physics). Thus, only one has to compare pairs of single scalar values between experiments on samples of the same anisotropic material from different sources, or on different samples from the same piece of anisotropic material.

Fortunately, what eventually came to the attention of Katz and Meunier was a series of papers by Bucur describing UWP measurements on a number of different soft and hard wood samples, treating each as having orthotropic symmetry [27, 28] comparable to the same symmetry found in young mammalian plexiform bone [26, 32]. Katz and Meunier then applied the same idea that selected volumes of wood could be treated as pseudo-crystals on the microstructural level. Thus, Bucur's data enabled Katz and Meunier to calculate both the shear and compressive, anisotropy factors for her wood samples, Table 4 in Katz and Meunier [30], and compare them with those they calculated for both plexiform and haversian bone, Table 1 in Katz and Meunier [30]. We have performed similar calculations for mechanical testing data performed under quasi-static conditions on woods [7] other than those used by Bucur in her MHz ultrasonic measurements [27, 28]. This additional calculation made it apparent that rather than just tabulating, as was done for Bucur's data [30], a plot of As* (%) versus Ac* (%) would highlight the correlations that indicate that both hard and soft woods exhibit the same type of material symmetry. Figure 3 shows the comparison in graphic form. This figure indeed shows that a "master curve" relating shear and compressive properties exists for living materials with similar microstructural symmetry, such as bone and wood. This "master curve" can be applied over a widerange of material

Table 1 Technical moduli for 11 woods measured by quasi-static techniques [33]

| All values are in Gpa	E_1	E_2	E_3	G_{13}	G_{12}	G_{23}	v_{13}	v_{12}	v_{32}	v_{23}	v_{31}	v_{21}
Balsa	3.38	0.05	0.16	0.18	0.12	0.02	0.23	0.49	0.67	0.23	0.02	0.01
Birch yellow	13.85	0.69	1.08	1.02	0.94	0.24	0.43	0.45	0.70	0.43	0.04	0.02
Douglas Fir (Interior North)	12.16	0.61	0.83	0.78	0.95	0.09	0.29	0.45	0.39	0.37	0.04	0.03
Spruce, Sitka	10.82	0.47	0.84	0.69	0.66	0.03	0.37	0.47	0.44	0.24	0.04	0.02
Sweet gum	11.30	0.56	1.30	1.01	0.69	0.24	0.32	0.40	0.68	0.31	0.04	0.02
Walnut, black	11.58	0.65	1.23	0.98	0.72	0.24	0.50	0.63	0.72	0.38	0.05	0.04
Yellow poplar	10.89	0.47	1.00	0.82	0.75	0.12	0.32	0.39	0.70	0.33	0.03	0.02
Douglas Fir (coast)	13.40	0.67	0.91	0.86	1.05	0.09	0.29	0.45	0.39	0.29	0.02	0.02
Mahogany, African	9.70	0.49	1.08	0.85	0.57	0.20	0.30	0.64	0.60	0.26	0.03	0.03
Mahogany, Honduras	10.30	0.66	1.10	0.68	0.89	0.29	0.31	0.53	0.60	0.33	0.03	0.03
Spruce, Engelmann	9.80	0.58	1.25	1.22	1.18	0.09	0.42	0.46	0.53	0.26	0.08	0.06
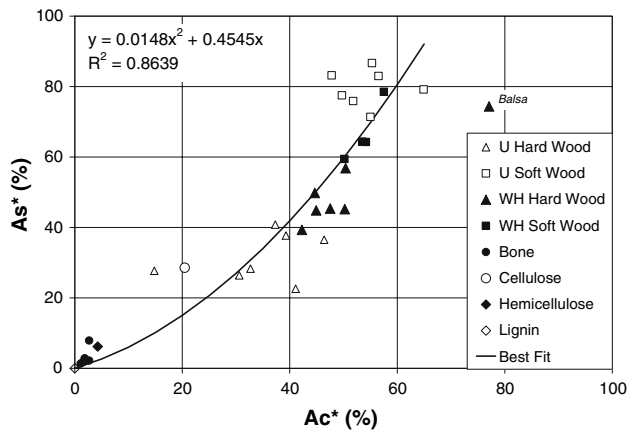
Fig. 3 Graph of shear, As* (%) vs. compressive, Ac* (%), scalar anisotropy factors for plexiform bone and soft and hard woods. U stands for ultrasonic measurements; WH stands for Wood Handbook Mechanical Testing measurements

properties whether obtained by ultrasonic wave propagation at MHz frequencies or mechanical testing techniques performed under quasi-static conditions. It is interesting to see that there is a clear separation between hard and soft woods that have the same material symmetry. The only exception is Balsa; the explanation may hinge on the fact that the anisotropic elastic properties of Balsa are much lower than either the hard or soft woods. If Ac* (%) and As* (%) are higher, then the degree of elastic anisotropy is higher. Our "master curve" shows the same type of quadratic behavior as shown in the curves for both cubic and hexagonal single crystals in Chung and Buessem paper [31].

The order of magnitude difference between the wood and the bone anisotropy, wood > bone, is clearly seen in these figures as well as the systematic differences in anisotropy between the hard and soft woods, i.e., soft wood > hard wood anisotropy. This again, yields insight into structure/property/function (SPF) relationships as the deformation response to load required for trees is significantly different than that required for bones as well as for soft woods relative to hard woods.

Results

In order to provide additional insight into the respective elastic properties of both hard and soft woods measured by UWP with the tensile measurements of wood by Keckes et al. [9], we have calculated the anisotropic technical moduli for both soft and hard woods derived from the technical moduli measured by traditional mechanical testing, i.e., quasi-static stress–strain measurements. The full set of nine technical moduli for 11 kinds of woods measured by quasi-static techniques (elastic ratios for various species (Forest Products Laboratory, Wood Handbook,

Chpt. 4: p 4–6, Wood Handbook 1999. U.S.D.A., Washington, DC) [33], is reproduced in Table 1 (E, Young's modulus; G, shear modulus; v, Poisson's ratio)).

The full matrix for technical moduli follows, Eq. 2:

$$[S_{ij}] = \begin{bmatrix} \frac{1}{E_1} & \frac{-v_{21}}{E_2} & \frac{-v_{31}}{E_3} & 0 & 0 & 0 \\ \frac{-v_{12}}{E_1} & \frac{1}{E_2} & \frac{-v_{32}}{E_3} & 0 & 0 & 0 \\ \frac{-v_{13}}{E_1} & \frac{-v_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \quad (2)$$

For an orthotropic material such as bone or wood, only 9 of the 12 nonzero terms in Eq. 2 are independent based on the symmetry of the S_{ij} tensor:

$$\frac{v_{12}}{E_1} = \frac{v_{21}}{E_2}; \quad \frac{v_{13}}{E_1} = \frac{v_{31}}{E_3}; \quad \frac{v_{23}}{E_2} = \frac{v_{32}}{E_3} \quad (3)$$

This further reduces for transverse isotropy to only five of the nonzero terms:

$$E_1 = E_2; \quad v_{12} = v_{21}; \quad v_{13} = v_{31} = v_{23} = v_{32};$$
$$G_{23} = G_{31}; \quad G_{12} = \frac{E_1}{2(1 + v_{12})} \quad (4)$$

The full C_{ij} matrix for an orthotropic material follows, Eq. 5:

$$[C_{ij}] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \quad (5)$$

The full set of nine C_{ij} for 11 kinds of woods, listed in Table 2, has been calculated, from Table 1 above. As the E_1 value of Mahogany–Honduras was unavailable, the value of 10.3 GPa given for Mahogany was used in the calculation.

Finally the C_{ij} in Table 2 have been used to calculate the Voigt (symbolized by a superscript V) and Reuss (symbolized by a subscript R) moduli and thence the values of As* (%) and Ac* (%), Table 3. In order to compare the relative anisotropies measured by mechanical testing [33], the values of As* (%) versus Ac* (%) from Table 3 have been added to Fig. 3 with those measured by UWP [27, 28].

Discussion

It is clear that measurement of the elastic properties of living tissues such as bone and wood are not identical from sample to sample or when made by different techniques.

Table 2 Elastic stiffness coefficients, C_{ij}, for 11 woods

All values are in Gpa	C_{11}	C_{22}	C_{33}	C_{44}	C_{55}	C_{66}	C_{12}	C_{13}	C_{23}
Balsa	3.43	0.06	0.19	0.12	0.18	0.02	0.05	0.09	0.04
Britch yellow	14.69	1.03	1.62	0.94	1.02	0.24	0.79	1.08	0.74
Douglas Fir (Interior North)	12.68	0.73	0.99	0.95	0.78	0.09	0.54	0.61	0.34
Spruce sitka	11.22	0.53	0.97	0.66	0.69	0.03	0.36	0.54	0.25
Sweet gum	11.75	0.74	1.70	0.69	1.01	0.24	0.47	0.78	0.53
Walnut black	12.94	0.97	1.84	0.72	0.98	0.24	1.02	1.40	0.75
Yellow poplar	11.26	0.62	1.34	0.75	0.82	0.12	0.41	0.63	0.45
Douglas Fir (coast)	13.70	0.77	1.04	1.05	0.86	0.09	0.42	0.44	0.31
Mahogony_African	10.21	0.60	1.32	0.57	0.85	0.20	0.50	0.63	0.37
Mahogony_Honduras	10.85	0.85	1.41	0.89	0.68	0.29	0.61	0.71	0.50
Spruce_Engelmann	10.86	0.72	1.56	1.18	1.22	0.09	0.75	1.15	0.43

Table 3 Voigt and Reuss moduli, and As* (%) and Ac* (%) for 11 woods

	K^{V} (GPa)	K_{R} (GPa)	G^{V} (GPa)	G_{R} (GPa)	Ac* (%)	As* (%)
Balsa	0.45	0.06	0.30	0.04	77.07	74.37
Birch yellow	2.51	0.96	1.42	0.48	44.66	49.80
Douglas Fir (Interior North)	1.93	0.59	1.22	0.27	53.48	64.37
Spruce sitka	1.67	0.45	1.05	0.13	57.50	78.50
Sweet gum	1.97	0.70	1.21	0.46	47.52	45.37
Walnut black	2.45	0.93	1.23	0.47	44.92	44.86
Yellow poplar	1.80	0.59	1.12	0.31	50.39	56.83
Douglas Fir (Coast)	1.98	0.59	1.36	0.29	54.18	64.28
Mahogany, African	1.68	0.56	1.03	0.39	50.23	45.21
Mahogany, Honduras	1.86	0.76	1.12	0.49	42.25	39.32
Spruce, Engelmann	1.98	0.66	1.22	0.31	50.18	59.46

Thus, it is not unexpected to find differences between the moduli measured by quasi-static mechanical testing with the high strain rate, high-frequency UWP techniques. Even so, one would expect that the elastic anisotropy measured in such tissues would be comparable as the differences are of scale and not of kind, even though the individual C_{ij} element may not have the same frequency dependency. An example of this is given for UWP measurements in bovine bone in Table 4, where extensional (bar) wave measurements (low MHz frequency) from Lipson and Katz [32] are compared with dilatational longitudinal wave measurements (high MHz frequency) of Katz et al. [26].

For additional discussions of the hierarchical structure/property relationships in bone see Currey [34] and the two chapters, # 12 & 13, by Lucchinetti [35, 36] in Cowin [37]; chapter 13 [36] in Cowin [37] is especially relevant to the studies of Keckes et al. [9] and Kretschmnann [18] as it analyzes dense bone tissue as a molecular composite.

This expectation, stated above, that the elastic anisotropy measured in viscoelastic tissues would be comparable over a wide range of measurement frequencies should also be true for both soft and hard wood tissues with orthotropic symmetry, as dispersion of moduli with frequency in wood can be expected to be less than that in bone. Figure 3, the plot of As* (%) versus Ac* (%) for bone, and both the UWP (at low MHz) and MT (quasi-static, i.e., at very low Hz) measurements of both soft and hard woods clearly illustrates this. The MT data fall into separate domains between the soft and hard woods almost exactly as do the UWP data. Balsa wood provides an interesting deviation as the lone outlier on the graph shown in Fig. 3. Clearly, this

Table 4 Ratios (%) of stiffness moduli in the transverse plane to that along the bone axis for bovine cortical bone

Experiment	Bone Type		
	Plexiform		Haversian
	Radial direction	Tangential direction	
Katz et al. [26]	52.3	64.5	65.7
Lipson and Katz [32]	58.0	71.8	64.5

must be due to the unusual softness of the tissue, affecting its structure/property relationship relative to those of both the soft and hard woods measured either by MT and/or UWP techniques.

The question of scale raised above is most important in developing any understanding of the material properties of complex hierarchical living materials such as wood and bone. This is evinced by a number of experiments directed at structure/property relationships in various woods at different scales, e.g., Hofstetter et al. [25], Kretschmann et al. [38], Cramer et al. [39]. Thus, this is also the reason for the experiment of Keckes et al. [9] and the associated comments by Kretschmann [18].

It is interesting to note that in the 1960s and 1970s there were a number of mechanical testing experiments to obtain the elastic symmetry of the lower size scale constituents of wood, e.g., lignin (isotropic) [40], hemicellulose, and cellulose (transverse isotropy, modeled on lignin) [41], effect of fiber orientation in the cell wall [42], and cell wall anisotropy [43].

The same calculations for the woods described above have been done for both the set of isotropic C_{ij} for lignin [40], and for the transverse isotropic C_{ij} for hemicellulose and cellulose [41][1]; these data are listed below in Table 5.

The values of As* (%) and Ac* (%) are plotted in Fig. 3. There is one concern with these C_{ij} for both hemicellulose and cellulose, as both C_{44} and C_{55} are given the same value as C_{66} based on scaling the lignin values. The values of $C_{66} = \frac{1}{2}(C_{11} - C_{12})$ for both hemicellulose and cellulose is consistent with elasticity theory. However, there is no a priori reason why both C_{44} and C_{55} should equal C_{66}. Clearly these values should be determined by direct experiments. It appears likely that these present values of C_{44} and C_{55} will not impact seriously on the values of As* (%) and Ac* (%). Therefore, we include them in Fig. 3 as they provide a picture of the effects of scale on the hierarchical anisotropic properties of wood. It points to the fact that understanding and modeling the properties of wood is a complex task as the symmetry changes with scale. Lignin is isotropic, hemicellulose and cellulose are transversely isotropic, while the cells and microstructure have orthotropic symmetry. It is only by using these two scalar anisotropy factors that we can place in perspective the full hierarchical effect of wood's constituent structures and its properties.

It is important to note that the C_{ij} are functions of frequency, and the functional dependence of each element need not be the same. Since dry wood has a lower tan delta

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[1] The transverse isotropic matrices for both cellulose (Eq. 3.1) and hemicellulose (Eq. 3.2) provided on page 39 in Fracture and Fatigue in Wood [20] are incorrect as the authors inadvertently transposed the values of C_{11} and C_{33} from Cave [43].

Table 5 Values of C_{ij}, Voigt and Reuss moduli and As* (%) and Ac* (%) for lignin [40] and hemicellulose and cellulose [41]

	C_{11} (GPa)	C_{22} (GPa)	C_{33} (GPa)	C_{44} (GPa)	C_{55} (GPa)	C_{66} (GPa)	C_{12} (GPa)	C_{13} (GPa)	C_{23} (GPa)	K^V (GPa)	K_R (GPa)	G^V (GPa)	G_R (GPa)	Ac* (%)	As* (%)
Cellulose	27.0	27.0	137.0	6.5	6.5	6.5	14.0	14.0	14.0	30.56	20.17	13.83	7.69	20.47	28.56
Hemicellulose	4.0	4.0	8.0	1.0	1.0	1.0	2.0	2.0	2.0	3.11	2.86	1.27	1.12	4.26	6.17
Lignin	4.0	4.0	4.0	1.0	1.0	1.0	2.0	2.0	2.0	2.67	2.67	1.00	1.00	0.00	0.00

than bone over most of the frequency range, the dispersion of moduli with frequency in wood can be expected to be less than that in bone.

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